Binary Black Holes and Globular Clusters

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Outline

- Broad-brush picture of evolution of BH subsystem
- Order of magnitude estimates
- Numerical results
  - Direct $N$-body of globular cluster
  - Introduction to Fewbody
  - Monte-Carlo of BH subsystem
  - Direct $N$-body of BH subsystem
Globular cluster, stars form

Heavy stars ($\gtrsim 20M\odot$) become BHs ($t \sim 10$ Myr)

BHs mass segregate into cluster core ($t \sim 100$ Myr)

BH system decouples dynamically and begins to evolve independently from the rest of the cluster

- if no primordial binaries, they are formed as BH system core collapses
- BHs do not interact with other cluster stars
- BH system destroys itself, ejecting tight binaries ($t \lesssim 1$ Gyr)
Assuming a reasonable IMF, \( N_{BH} \sim 10^{-4} N \)

BH birth kicks unknown, but perhaps
\[
\nu_{BH} \approx \frac{m_{NS}}{m_{BH}} \nu_{NS} \ll \nu_{esc}
\]
Fryer & Kalogera (2001)

Mass segregation timescale: \( t_{ms} \approx \frac{\langle m \rangle}{m_{BH}} t_{rh} \approx 100 \text{ Myr} \)
Fregeau, Joshi, Portegies Zwart, & Rasio (2002)

Spitzer mass stratification instability:
\[
\left( \frac{M_{BH}}{M_{clus}} \right) \left( \frac{m_{BH}}{\langle m \rangle} \right)^x \approx 10^{-4} \left( \frac{m_{BH}}{\langle m \rangle} \right)^{x+1} \approx \begin{cases} 0.21 \\ 3.4 \end{cases} \Rightarrow \begin{cases} 0.16 \\ 0.32 \end{cases}
\]
Watters, Joshi, & Rasio (2000)

Relaxation timescale of BH system:
\[
t_{r,BH} \approx 1 \text{ Myr} \left( \frac{\sigma}{10 \text{ km/s}} \right)^3 \left( \frac{m_{BH}}{15 M_\odot} \right)^{-1} \left( \frac{\rho}{10^5 M_\odot/\text{pc}^3} \right)^{-1}
\]
Order of magnitude estimates (cont’d)

- **Upper limit on sub-cluster evaporation time:**
  \[ t_{\text{evap}} \lesssim 10 \times 300 t_{\text{rh,BH}} \approx 1 \text{ Gyr} \]
  Kalogera, King, & Rasio (2003)

- **Properties of ejected binaries**
  - Thermal eccentricity distribution: \[ dP(e)/de = 2e \]
  - Semimajor axis: \[ E_b \gtrsim 10^3 kT, \text{ so } a \lesssim 0.1 \text{ AU} \]
  Hut, McMillan, & Romani (1992)

- **GW merger timescale:**
  \[ 300 \text{ Myr} \left( \frac{\mu M^2}{M_\odot^3} \right)^{-1} \left( \frac{a}{R_\odot} \right)^4 (1 - e^2)^{7/2} \sim 1 \text{ Gyr} \]
  Peters (1964)

- Performed direct $N$-body simulations of $\sim 2 - 4 \times 10^3$ total stars with $\sim 20 - 40$ primordial BHs of mass $10M_\odot$

- Of 204 total BHs in 9 simulations:
  - 62 ($\sim 30\%$) ejected in binaries
  - 124 ($\sim 61\%$) ejected as single BHs
  - 1 ($\sim 1\%$) ejected in binary with low-mass companion
  - 17 ($\sim 8\%$) retained by cluster

- Ejected with $E_b \sim 10^3 - 10^4 kT$

- Roughly thermal eccentricity distribution, with large $e$ slightly overrepresented

- Ejected within first few Gyr
Take $a$, $e$ distributions to be representative

Assume density of galaxies and star clusters in universe

Obtain rate $R = 7.5 \times 10^{-8} h^3 \text{yr}^{-1} \text{Mpc}^{-3}$

Estimate LIGO detection rate of $1.7 h^3 \text{yr}^{-1}$
- Mergers due to Kozai resonance in triples

- Formation of IMBH by runaway growth via mergers during BH subsystem collapse

- Stable BH system remaining in cluster after BH subsystem disrupts may be dynamically important
  Colpi, Possenti, & Gualandris (2002)

- Run-away growth before many BHs form?
Brief introduction to Fewbody (www.mit.edu/~fregeau)

- Designed for small-$N$ dynamics
- Uses adaptive integrator, with optional pairwise K-S regularization \( \text{Mikkola (1985)} \)
- Binary trees for classification
- Mardling triple stability criterion to determine dynamical stability of full hierarchies \( \text{Mardling & Aarseth (2001)} \)
- Automatic calculation termination
- Binary trees for isolating unperturbed hierarchies from integrator
- Physical collisions in the sticky star approximation:
  \[
  R_{\text{merger}} = f_{\exp}(R_1 + R_2)
  \]
Movies

- Binary-single
- Binary-binary
- Triple-binary
- Cluster
Treat BH system via Monte-Carlo
Do binary interactions with *Fewbody*
Between interactions, evolve binaries using Peters equations
Check for Kozai-induced mergers in any stable triples formed
Initial conditions

- **BHs**
  - $N_{\text{BH}} = 1024$
  - $dN/dM \propto M^{-2}$
  - $M \in [5, 15] M_\odot$
  - 100% binary fraction, all hard binaries
  - $p(q) \propto q$
  - $p(a) \propto a^{-1}$
  - $a \in [10, 10^4] R_\odot$
Cluster

- Core-halo model
- \( n_c = 10^5 \text{ pc}^{-3} \)
- \( v_{\text{esc,core}} = 30 \text{ km/s} \)
- \( v_{\text{esc,clus}} = 60 \text{ km/s} \)
- \( t_{\text{rh}} = 1 \text{ Gyr} \)
- \( t_{\text{ms}} = \frac{\langle m \rangle}{m_{\text{BH}}} t_{\text{rh}} \)
Run-away growth

**Largest Black Hole Mass in Cluster**

- **Largest BH**
- **Average BH Mass**

**Axes:**
- **Mass [M_☉]**
- **Time [yrs]**

![Graph showing the growth of black hole mass over time.](image-url)
Relevance for LIGO

Chirp Mass of Merged Black Hole Binaries

- In Core
- Ejected

Time [yrs]

Chirp Mass [M_☉]

$10^6$  $10^7$  $10^8$  $10^9$  $10^{10}$
- Use *Fewbody* to do BH subsystem
- Add potential of globular cluster (static Plummer model for now)
- Add dynamical friction on BHs due to cluster stars
Initial conditions

- 25 BHs of mass $20 M_\odot$
- Cluster mass of $10^5 M_\odot$
- Cluster core density of $10^5 M_\odot/pc^3$
- BH positions and velocities drawn from cluster distribution function, out to a limiting radius of $r_h$, to mimic primordial mass segregation
Direct $N$-body for BHs (cont’d)

- Of 775 total BHs in 31 simulations:
  - $\sim 40\%$ ejected as binary BHs
  - $\sim 45\%$ ejected as single BHs
  - $\sim 15\%$ retained by cluster, in the form of triples, etc.

- Roughly thermal eccentricity distribution

- Ejected before $\lesssim 1\text{Gyr}$
Distribution of eccentricities

dP/ de vs. e

- p.20/23
Distribution of observable merger times

![Graph showing the distribution of observable merger times.](image)
What’s left?

- Usually a stable hierarchical triple (∼ 80% of the time)
- Triples retained in cluster core, never ejected
- Outer binary is wide ($a_{\text{out}} \sim 10^4 \text{ AU}$), so triple is likely to be disrupted quickly by field stars
- BH subsystem evolves quickly ($\sim 1$ Gyr)
- Equipartition between BHs and cluster stars not reached
- BH binaries ejected early ($\lesssim 1$ Gyr) with short ($\sim 1$ Gyr) merger timescales
- Runaway merger scenario plausible, even without seed BH