Simulation of coalescing binary neutron stars: Current Status and Issues

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Three stages of “Last Three minutes”

Post-Newtonian

Inspiral
- $f \sim 10 \rightarrow \sim 500$ Hz
- $r \sim 700 \rightarrow \sim 40$ km
- Post—Newtonian + point particle
- $r \gg R, \ Tgw \gg P$

Intermediate
- $f \sim$ several 100 Hz
- $r \sim 3R \sim 30 \rightarrow 45$ km
- Quasi-equilibrium
- $Tgw \gg P$

Merger
- $f > 1$ kHz
- $r < 30$ km
- Numerical Relativity

This talk

BH or NS
Expected gravitational waveforms

- Inspiral = PN templates
- Done up to 2.5 PN

Late-inspiral & Merger = Numerical relativity

This talk
Procedures in GR hydro simulations

1. Give a realistic initial condition at intermediate phase (quasiequilibrium)
2. Solve the Einstein evolution equations
3. Impose gauge conditions
4. Solve the GR hydrodynamic equations
5. Incorporate realistic EOS, micro physics (hopefully)
6. Extract gravitational waves in the wave zone
   - Prepare a large computational domain
7. Determine apparent horizons
8. BH excision or other techniques when BHs are formed
Current Status

1. Initial condition: Give a GR quasiequilibrium in the conformal flatness approximation
   → good, but no GW at t=0;
   To be improved for accurate computation of GW
2-a. Time slice = Maximal slice or dynamical slice (OK)
2-b. Spatial gauge = Dynamical gauge (OK)
3. Einstein’s evolution equations = BSSN formalism
   (OK in absence of BH)
4. Hydrodynamic equations = High-resolution shock capturing scheme (OK)
5. EOS = Ideal fluid, simple → to be improved
6. Wave extraction = Gauge invariant extraction in a local wave zone (fair) → to a distant wave zone
7. Apparent horizon = easy to determine (OK)
8. BH Excision → Not yet; LAZARUS – could be done

Qualitative study is feasible to BH formation.
Our latest implementation

1. **GR** : BSSN (modified)
   (latest version = Shibata et al. PRD68, 084020, 2003)
2. **Gauge** : Maximal slicing ($K = 0$) + Dynamical gauge
3. **Hydro** : High-resolution shock-capturing scheme
   (Roe-type method with 3rd-order PPM interpolation)
4. **Initial condition** = Conformal flatness approximation
5. **Typical grid size** : $633 \times 633 \times 317$
   (~ 200 GByte memory, ~ 100 CPU hour)
   Using FACOM-VPP5000 @NAOJ
   (Max size so far $697 \times 697 \times 349$)
6. **EOS** : $P = (\Gamma \Pi) \rho \varepsilon$
Some latest results

EOS: Initial \( P = K \rho^\Gamma, \; \Gamma = 2; \; K = 1.535d5 \) cgs

\[
M = 1.4 \, M_{\text{solar}} \rightarrow R = 14.8 \, \text{km}
\]

\[
1.6 \, M_{\text{solar}} \rightarrow R = 13.3 \, \text{km}
\]

Max. mass for spherical case = 1.68 \( M_{\text{solar}} \)

I here show animations for

(a) 1.4 – 1.4,

(b) 1.33 – 1.46,

(c) 1.52 – 1.52,

(d) 1.4 – 1.6 \( M_{\text{solar}} \)

(Shibata et al. PRD 68, 084020, 2003)
Oscillating hypermassive neutron stars are formed for unequal mass \(1.33-1.46\) and equal mass \(1.4-1.4\). Not crash. We artificially stopped simulation.
1.4 – 1.4 $M_{\odot}$ solar case: final snapshot

Massive toroidal neutron star is formed

(slightly elliptical)
Formed Massive toroidal NS is differentially and rapidly rotating

Angular velocity

Kepler angular velocity at stellar surface
Comparison between equal and unequal mass mergers

1.33—1.46:
Massive NS + disk

Unequal-mass case
Mass ratio \( \sim 0.901 \)

1.4—1.4:
Massive NS

Equal-mass case
Black hole formation case: 1.52—1.52

Equal-mass case

Mass for $r > 3M \sim 0.2\%$

Apparent horizon
Disk mass for unequal-mass merger

1.45—1.55, Mass ratio 0.925  
Mass for $r > 3M$  
$\sim 2\%$

1.4—1.6, Mass ratio 0.855  
Mass for $r > 3M$  
$\sim 4\%$
1. Equal – mass cases
   - Low mass cases
     Hypermassive neutron stars
     of nonaxisymmetric & quasiradial oscillations.
   - High mass cases
     Direct formation of Black holes
     with very small disk mass

2. Unequal – mass cases (mass ratio ~ 0.9)
   - Disks of mass ~ several percents of total mass
     $\Rightarrow$ BH(NS) + Disk
Gravitational waves for NS formation

$1.4M_\odot - 1.4M_\odot, R = 15\text{km}$

$l=m=2$ mode

$l=2, m=0$ mode

Nonaxisymmetric oscillation of NS

Quasi-radial oscillation of NS

Stationary quadrupole

$f \sim 2.2\text{kHz}$

$f \sim 0.7\text{kHz}$
Gravitational waves from unequal-mass merger to NS formation

$l=m=2$ mode

$l=2, m=0$ mode

$1.46M - 1.33M$

$f \sim 2.3 \text{kHz}$

Nonaxisymmetric oscillation of NS

$f \sim 0.7 \text{kHz}$

Quasi-radial oscillation of NS

Stationary quadrupole
Fourier spectrum in NS formation

- Inspiral wave
  - $\sim 1.5\text{kHz}$
  - $\sim 2.2\text{kHz}$ (equal mass)
  - $\sim 2.3\text{kHz}$ (unequal mass, Mass ratio=0.9)
  - $\sim 0.7\text{kHz}$

- Frequency also depends on EOS.

- $|R_{lm}/r/M_0|$
Computation of mass and angular momentum

Check of the conservation

**GW**

Computational domain

\[ M', J' \]

**GW**

Whole region

\[ M = M_0 \]
\[ J = J_0 \]

\[ M_0 - E_{GW} = M' \]
\[ J_0 - J_{GW} = J' \]

should be satisfied
Radiation reaction: OK within ~ 1%

NS formation: equal mass

Solid curves: computed from data sets in finite domain.
Dotted curves: computed from fluxes of gravitational waves

BH formation: unequal mass

Mass energy

Angular mom.
Issues for quantitatively accurate computation

1. Initial condition (quasiequilibrium):
   Beyond the conformal flatness approximation
   to include gravitational waves at $t=0$
   → Probably the best way is to solve full sets of
     Einstein’s equations for a quasiequilibrium

2. Excision or LAZARUS for computation of quasinormal mode of BH (but too high frequency ~ 6—7 kHz; might be irrelevant to LIGO/TAMA/GEO/VIRGO)

3. Incorporate realistic micro physics:
   First step = incorporate realistic EOSs;
   We should learn from supernova community

4. Wave extraction in the wave zone:
   Prepare a large computational domain
   → AMR/FMR or Hypercomputers
Gravitational waves in BH formation

\[ 1.52M_\odot - 1.52M_\odot, R = 14\text{km} \]

\[ R_{2:2} \propto m_2/M_0 \]

Not very accurate

\( l=m=2 \) mode

BH QNM

Ringing

\( f \sim 6-7\text{kHz} \)

CRASH
Energy curve of circular orbits

$E / M = \frac{v^2}{(M \Omega)^{2/3}} \sim M / R$

1.4–1.4$M_\odot$, $R=14.8$km

- Point-particle Approximation Works well
- 2PN point particle
- Quasiequilibrium

Distant orbits

Tidal effect becomes important
~ 3.3–3.4 $R$

Initial cond. should be given here.
Issues for computational resources

Match to PN/Point particle waveforms

- Tidal effect is important for \( r < 3.5R \approx 9-12M \)
- Emission time scale should be \( > \sim 5P \) for quasiequilibrium

\[
\frac{t_{\text{coalesce}}}{P_{\text{orbit}}} \approx 4.4 \left( \frac{r}{3.5R} \right)^{5/2} \left( \frac{R}{12.4\text{km}} \right)^{5/2} \left( \frac{M}{2.8M_\odot} \right)^{-5/2} \left( \frac{M}{4\mu} \right)
\]

\[
\left| \frac{v_r}{v_\varphi} \right| \approx 9 \times 10^{-3} \left( \frac{3.5R}{r} \right)^{5/2} \left( \frac{12.4\text{km}}{R} \right)^{5/2} \left( \frac{M}{2.8M_\odot} \right)^{5/2} \left( \frac{4\mu}{M} \right)
\]

\[
f \approx 680\text{Hz} \left( \frac{3.5R}{r} \right)^{3/2} \left( \frac{12.4\text{km}}{R} \right)^{3/2} \left( \frac{M}{2.8M_\odot} \right)^{1/2}
\]

\[\Rightarrow \lambda \approx 440\text{km}, \ \Delta x < 0.5\text{km}, \ L > \lambda \Rightarrow N = 2 \frac{L}{\Delta x} > 1700 ! \Rightarrow \sim 5\text{TBytes}\]

Very large computational domain is necessary

\[\Rightarrow \text{Hyper-Computers or Adaptive (or fixed) mesh refinement}\]
Summary: History of code development

• 1: Make a code which runs anyhow stably (do not care accuracy)
• 2: Improve the code which can provide a qualitatively correct result; care accuracy somewhat (say we admit an error of ~10%)
• 3: Improve the code gradually getting qualitatively new results which can be obtained only by an improved code

★ 4: Goal: Make a code which provides a quantitatively accurate result.

We are here.

Similar to construction of detectors in some sense
Animations

- http://esa.c.u-tokyo.ac.jp/~shibata/anim.html