Before Gravitational Waves: Issues in Simulations of Core Collapse Supernovae

Anthony Mezzacappa

ORNL/TSI
Supernova Sources of Gravitational Waves

- Instabilities
  - Proto-Neutron Star Instabilities
    - Convection
    - Neutron Fingers
    - Neutrino-Driven Convection

- Rotation
  - With and Without Magnetic Fields

- Combination

Increasing Ignorance

Rotation with Magnetic Fields
Rotation sans Magnetic Fields
Neutrino-Driven Convection
Proto-Neutron Star Neutron Fingers
Proto-Neutron Star Convection
Previous Efforts

Convection:
- PNS: 100-1000 Hz
- ND: 3-100 Hz
- Energy Emitted 2-3 Orders of Magnitude Smaller in 3D PNS

Rotation (Axisymmetry): 500-1000 Hz

Asymmetric Collapse/Anisotropic Neutrino Emission: 10-500 Hz
- Burrows and Hayes, *PRL* 76, 352 (1996)

Rotation (Non-Axisymmetric):
- No considerable enhancement of gravitational radiation!

Starting Point and Paradigm

Presupernova Structure
representative of a 15 $M_\odot$ star

Core Collapse and Explosion

1. 3x10^3 cm
2. 10^8 cm
3. O, Ne, Mg
4. Earth
5. O, Ne, Mg
6. "Si"
7. "Fe"

1. H, He
2. He
3. C, O
4. He
5. H, He
6. "Si"
7. "Fe"

neutrinos
shock
Essence of an Old Paradigm

Need Boltzmann Solution

Decrease with Anisotropy

Janka (2001)
Burrows and Goshy (1993)
Boltzmann transport results in significant quantitative changes in supernova models.

New nuclear physics and/or multi-D effects necessary ingredients in the explosion mechanism.
Ledoux Criterion *sans* Transport Effects:

$$\left( \frac{\partial \rho}{\partial \ln Y_l} \right)_{s,P} \left( \frac{\partial \ln Y_l}{\partial r} \right) + \left( \frac{\partial \rho}{\partial \ln s} \right)_{Y_l,P} \left( \frac{\partial \ln s}{\partial r} \right) > 0$$

*Negative entropy and lepton fraction gradients are destabilizing.*
Proto-Neutron Star Convection


Ray-By-Ray Transport GR
Mezzacappa et al. 1998

Spherically Symmetric Transport

Proto-Neutron Star Convection
Proto-Neutron Star Convection

\[ \dot{\nu} = \frac{g}{\rho} \alpha_s \theta_s \]

\[ \dot{\theta}_s = -\frac{\theta_s}{\tau_s} - \frac{d\bar{s}}{dr} \nu \]

\[ \theta_s = s - \bar{s} \]

\[ \alpha_s \equiv -\left( \frac{\partial \rho}{\partial s} \right)_{P,Y_1} \]

\[ \frac{1}{\tau} = \left( \frac{1}{\tau_{BV}}^2 + \frac{1}{4\tau_s^2} \right)^{1/2} \]

\[ -\frac{1}{2\tau_s} \rightarrow \frac{\tau_s}{\tau_{BV}} \frac{1}{\tau_{BV}} \]

\[ \nu_{c,\text{asymptotic}} \rightarrow \frac{\tau_s}{\tau_{BV}} \left( \nu_{c,\text{asymptotic}} \right)_{\text{no transport}} \]

\[ \left( \nu_{c,\text{radial/ angular}} \right)_{\text{no transport}} \sim 10^{8-9} \text{ cm/s} \]

\[ \left( \nu_{c,\text{radial/ angular}} \right)_{\text{with transport}} \sim 10^6 \text{ cm/s} \]
**Neutron Fingers**

Equations governing motion of fluid element:

\[
\begin{align*}
\dot{\theta}_s &= \Sigma_s \theta_s - \Sigma Y_l \theta_{Y_l} - \frac{d\bar{s}}{dz} \cdot \theta_s \\
\dot{\theta}_{Y_l} &= Y_s \theta_s - Y_{Y_l} \theta_{Y_l} - \frac{d\bar{Y}_l}{dz} \cdot \theta_{Y_l} \\
\rho \ddot{z} &= -g \left( \frac{\partial \rho}{\partial s} \right)_{P,Y_l} \theta_s - g \left( \frac{\partial \rho}{\partial Y_l} \right)_{P,s} \theta_{Y_l}
\end{align*}
\]

Solutions:

\[
\begin{align*}
Ae^{\alpha_1 t} + Be^{\alpha_2 t} + Ce^{\alpha_3 t} \\
Ae^{\alpha_1 t} + Be^{(\alpha_2 + i\beta) t} + Ce^{(\alpha_2 - i\beta) t}
\end{align*}
\]

\[
\alpha_i = f(s, Y_l, \alpha_s, \alpha_{Y_l}, \Sigma s, Y_{Y_l}, \frac{d\bar{s}}{dz}, \frac{d\bar{Y}_l}{dz})
\]

**Bruenn and Dineva (1996)**
Neutron Fingers

Wilson and Mayle (1993) - EoS Dependent

Bruenn and Dineva (1996)
**Neutrino Driven Convection**

- Need fully 3D models.
- Rotation will alter flow.
  - **Know very little about this interaction.**
- This is a radiation hydrodynamics problem.
  - **Transport powers explosion.**
  - Convection depends on evolution of the gain region.

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Herant et al. (1994)
Mezzacappa et al. (1998)
Swesty (1998)
Newtonian vs. GR Hydrodynamics

25 Solar Mass Model, 300 ms after Core Bounce

Newtonian Simulation

GR Simulation

Shock

Gain Radius

Cooling

Heating

Matter Flow

Neutrinos
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Rotation with Magnetic Fields
Rotation sans Magnetic Fields
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Essentially hydrodynamics studies.

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1. neutrinos
2. shock
6. 7.
Essence of an Old Paradigm

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\[ \dot{E} = \frac{X_n}{\lambda^2} \frac{L_{ve}}{4\pi r^2} \langle E_{ve} \rangle \left( \frac{1}{F} \right) + \frac{X_p}{\lambda^2} \frac{L_{ve}}{4\pi r^2} \langle E_{ve}^2 \rangle \left( \frac{1}{F} \right) \]

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Ray-By-Ray Transport GR
Proto-Neutron Star Convection

Mezzacappa et al. 1998

Spherically Symmetric Transport
Newtonian
\[ \nu = \frac{g}{\rho} \alpha_s \theta_s \]

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Dud

Explosion
Newtonian vs. GR Hydrodynamics

25 Solar Mass Model, 300 ms after Core Bounce

Newtonian Simulation

- Shock
- Gain Radius
- Cooling
- Heating
- Matter Flow
- Neutrinos

GR Simulation

- Shock
- Gain Radius
- Cooling
- Heating
- Matter Flow
- Neutrinos
Rotation/Magnetic Fields: Jets/New Paradigm?

★ Neutrino-driven models with convection and rotation.
  Fryer and Heger (2000)

★ MHD-driven models with rotation and magnetic fields.
  LeBlanc and Wilson (1979)
  Symbalisty (1984)
  Khokhlov, Hoeflich, Oran, Wheeler, Wang, and Chtchelkanova (1999)

➤ Paradigm shift?
➤ Multiple mechanisms?

★ “Collapsar” models driven by neutrinos, MHD effects, or both.
  MacFadyen and Woosley (1999)

➤ Multiple mechanisms?
Precollapse models are improving.

Nonrotating Models
- Umeda, Nomoto, and Nakamura (2000)

Rotating Models

Mechanisms for angular momentum transport.

“...the distribution of angular momentum in the star at onset of core collapse strongly reflects its recent convective structure.”
Nuclear and Weak Interaction Physics Needs

Thomas Fermi (Classical)
Classical treatment of many-body problem.

Hartree-Fock
Lowest order solution to the quantum mechanical many-body problem.

Shell Model Monte Carlo
Shell Model Diagonalization
Advanced solutions to the many body problem.

Bloch-Horowitz
Solve “exact” many-body problem.

High-Density EoS
Nuclear Matter Opacities
Ensembles
Density of States
e-capture
ν-nucleus
β-decay
Time
Inner core mass proportional to square of mean electron fraction. 
~10% change in inner core mass dynamically significant.

Langanke and Martinez-Pinedo, NPA 673, 481 (2000)
Past Approximations:
Degeneracy and Relativity
Composition (allowed degrees of freedom)
Baryon-Baryon Interactions


*Interacting matter of arbitrary degeneracy and relativity.*
*Accounted for in-medium mass and energy shifts at Hartree-Fock level.*
*Multiple-component matter (e.g., hyperons).*
*Additional interaction channels.*


Correlations.
*Accounted for interactions at the RPA level.*

Raffelt and Seckel (1995)
Janka et al. (1996)
Hannestadt and Raffelt (1999)

Burrows and Sawyer, PRC 58, 554 (1998)
Burrows and Sawyer, PRC 59, 510 (1999)
Challenges and Requirements

- Develop a standard model of core collapse supernovae.
  - 3D.
  - Accurate multigroup neutrino transport.
  - Realistic nuclear and weak interaction physics.
  - GR.
  - MHD.

- Scalable algorithms for radiation hydrodynamics (MHD, MRHD).
  - Scalable algorithms for the solution of large, sparse linear systems/large eigenvalue problems.

- Parallel programming issues.
- Software engineering issues.
- Collaborative visualization.

- Requires an interdisciplinary team effort: astrophysicists, nuclear physicists, applied mathematicians, computer scientists.
Scientific Discovery through Advanced Computing

Office of Science
U.S. Department of Energy

March 24, 2000

http://www.phy.orl.gov/tsi/
Goal: Develop a Standard Model of Core Collapse Supernovae

- Successful model for the explosion mechanism(s)
- Reproduce supernova phenomenology

Neutrino, Gravitational Wave, Gamma Ray Signatures

- Neutron star kicks
- Gamma ray burst association
Investigator Team

Linear System/Eigenvalue Problem Solution
Algorithms for Radiation Transport and Nuclear Structure Computation
- Dongarra (UT, ORNL)
- Saied (UIUC, NCSA)
- Saylor (UIUC, NCSA)

Radiation Transport/Radiation Hydrodynamics
- Blondin (NC State)
- Bruenn (FAU)
- Hayes (UCSD)
- Mezzacappa (ORNL)
- Swesty (SUNYSB)

Nuclear Structure Computations for EOS and Neutrino-Nucleus/Nucleon Interactions
- Dean (ORNL, UT)
- Fuller (UCSD)
- Haxton (INT, Washington)
- Lattimer (SUNYSB)
- Prakash (SUNYSB)
- Strayer (ORNL, UT)

Supernova Science
- Blondin
- Bruenn
- Fuller
- Haxton
- Hayes
- Lattimer
- Meyer (Clemson)
- Mezzacappa
- Swesty

Cross-Cutting Team
- Long-Term Collaborations
- Structured like SciDAC

Visualization
- Baker (NCSA)
- Toedte (ORNL)

DATA CCA PERC TSTT
Supernova Simulation Timeline

- **Start**: 2D Ray-by-Ray (MGBT) Newtonian
- **Year 1**: 2D Ray-by-Ray (MGBT) Approximate GR
  - 3D Ray-by-Ray (MGBT) Newtonian
- **Year 2**: 3D Ray-by-Ray (MGBT) Approximate GR
  - 2D MGFLD Newtonian
- **Year 3**: 2D MGFLD Approximate GR
  - 3D MGFLD Newtonian
  - 2D MGBT Newtonian
  - GW signatures from convection, rotation. 86: B fields.
Summary

Gravitational Waves from Proto-Neutron Star Instabilities (Convection, Neutron Fingers)
Requirements for Reliable Waveforms: 3D Radiation Hydrodynamics (Neutrino-Matter Interactions Important)

Gravitational Waves from Postshock 3D Flow (Neutrino-Driven Convection, Shock Induced Vorticity and Turbulence)
Requirements: 3D Radiation Hydrodynamics (Neutrino Flux Determines “Boundary Conditions”)

Gravitational Waves from Rotation/Combination of Rotation and Convection
Requirements: 3D Precollapse Models
3D Supernova Models

Gravitational Waves with B Fields
Requirements: 3D MRHD

TSI is committed to a concerted effort to compute reliable waveforms in all of the above cases, with first results available within the next few years for GWs from instabilities. (GWs from rotation will remain uncertain.)