CONVERGENCE IN NUMERICAL CALCULATIONS OF RADIATION REACTION EFFECTS.

(AND OTHER TOPICS)

BERNARD WHITING

UNIVERSITY OF FLORIDA

WORK WITH:

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R.R. COMMUNITY)
Real Title:
Scrap left over from focus workshop and previous Capra 5 talks.

Real Topics:
- \( L=0 \) & \( L=1 \) modes in Kerr.
- Convergence and smoothness.
- Debye potential & Weyl curvature.
- Time vs frequency domain.
- Schwarzschild & Kerr.
- Adding in sources.
$l=0 \& l=1$ Modes in Kerr

Real Issues:

- $\delta = \? \quad \text{mode sum}$
- $\text{tau} = \? \quad \text{mode sum}$

$\Rightarrow$ Tensors on Sphere?

RW:

$$\omega_0 = (\omega_0 + \frac{i}{\omega_0} \omega_+ \lambda) \ \ Y_{lm}$$

$$\omega_+ = Y_{lm}$$

CHRE:

$$\lambda_{lm} = \lambda^{+\perp^{+\perp}} S_{lm} + \ldots$$
$$\lambda_{lm} = -S_{lm}$$

* $\Lambda_{lm} = \Lambda_{lm}$ missing in rad. 5.

Know:

$\delta m$, $\delta q$, translations, rotations

But ans. sep. req. freq., domain
**Calculating & Fitting.**

- $A, B, D$ calculated to machine precision (large relative to $F^n$)
- $E_2 - E_4$ calculated to fitting accuracy. (Increasing relative error: $10^{-6} - 10^{-1}$)

<table>
<thead>
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<tbody>
<tr>
<td>$A$</td>
<td>$10^{-12}$</td>
<td>$10^{-16}$</td>
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<tr>
<td>$B$</td>
<td>$10^{-16}$</td>
<td>$10^{-18}$</td>
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<tr>
<td>$D$</td>
<td>$10^{-16}$</td>
<td>$10^{-22}$</td>
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<tr>
<td>$E_i$ (tail)</td>
<td>$10^{-6}$</td>
<td>$10^{-15}$</td>
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<tr>
<td>$E_i''$</td>
<td>$10^{-4}$</td>
<td>$10^{-15}$</td>
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<tr>
<td>$E_i'''$</td>
<td>$10^{-2}$</td>
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<tr>
<td>$E_i''''$</td>
<td>$10^{-1}$</td>
<td>$10^{-18}$</td>
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$R = 10 \mu m$

5 parameters fit for 12 points
$R = 10 \mu m$

5 parameters Fit 12 points
**CONVERGENCE AND SMOOTHNESS.**

\[(L^2 - 1)^{-1} \approx L^{-2}\]

\[\|\hat{\sigma} - \theta^{1/2}\| \approx 10^{1/2}\]

\[C^0 \text{ on sphere } \rightarrow C^0 \text{ at equator.}\]

In both cases **differentiability improves with increased fall-off**

\[|\hat{\sigma} - \theta^{1/2}| - |\theta^{1/2}| \sim 10^{3}\]

(*1-1 relation**!!*)

**Aside:** **Exactly same issue arises in data analysis.**
DEBYE POTENTIALS

\[ l_{\mu\nu} = D_{\mu\nu}(\Phi_0) \]

\[ \Rightarrow \Phi_{0\mu} = 0^+(\Phi_0). \]

SCHWARZSCHILD VACUUM

\[ \Phi_0(4_1) \quad \text{CAPRA 4} \]

\[ \Phi_0(4_0) \quad \text{FOCUS WORKSHOP, L CAPRA 5 (A.O.)} \]

USES TIME DOMAIN (OF 2\textsuperscript{nd} ORDER)

USES ANGULAR SEPARATION

WEYL APPEARS AS SOURCE FOR \( \Phi_0 \)

FIND HOMOGENEOUS SOLUTIONS ARE ALGEBRAICALLY SPECIAL.
SOURCES

- See Wald
- Start with Schwarzschild
  (Have result for metric
   already, but 'gauge' issues remain)
- In progress,

KERR

- Characterize a.s. without sep.
- Use $y_0$ and $y_4$ equations.
- Perhaps, start with potential.

- Hope springs eternal (almost!)
The Green's function for the Teukolsky equation (taking into account the known factorized form of the Green's function for the Schwarzschild's argument) reverse the steps of Christodoulou's argument. Initial results are rigorously established, one may Erwin Schrödinger perturbations of Kerr. Now let the first order factorization of the Green's function for the Teukolsky equation be obtained by positing that a factorized form

\[ \psi = \psi_0 + \phi \]

is a solution of the Teukolsky equation for \( \phi \). It is worth noting that Christodoulou's...