Loss cone: past, present and future

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Coalescence of compact objects with SMBH

- Galaxies tend to have "*cuspy*", non-isothermal centres.

  Rising density profile.

  $$\lim_{r\to 0} \rho \propto r^{-\gamma}$$

  $\gamma = -3/2 - p$, $p \in \{-1, +1\}$ (QHS '95)

- Central SMBH are near ubiquitous, with $M_{BH} \sim M_{Gal-Sph} \propto \sigma^n_{gal}$

  Rising velocity dispersion $\lim_{r\to 0} \sigma(r) \propto r^{-1/2}$,

  SMBH induced Keplerian increase.

  Velocity anisotropy $\beta(r)$, flattening/triaxiality (?) of density profile.

- Relaxation time $t_r \propto \sigma^3(r)/\rho(r)$, constant for $p = 0$.

  Most relevant galaxies are *not* relaxed in the centre.

- Normal stellar population (?). Compact remnants can coalesce with SMBH through gravitational radiation or direct capture.

  $f_{WD} \sim 0.1 - 0.3$, $f_{NS} \sim 10^{-3} f_{kick}$, $f_{BH} \sim 10^{-4}$

- Coalescence when scattering or diffusion in discrete stellar population puts object on radial orbit which shrinks through emission of gravitational radiation faster than it can be scattered to higher angular momentum orbits
- $h_c \approx 4 \times 10^{-24} \frac{1}{d/\mathrm{Gpc}} \frac{m}{M_\odot} \left(\frac{M}{10^6 M_\odot} \right)^{2/3} \left(\frac{10^4 \ \mathrm{sec}}{P_{orb}} \right)^{2/3}$
- *loss cone* - $\theta_{lc}(r)$ - by definition, where timescale for coalescence through gravitational radiation emission is shorter than timescale to diffuse out of loss cone through random walk in J.

  rate is sensitive to number density profile of stellar population in the center of galaxies, $\rho \propto r^{-3/2-p}$, to mass function and to anisotropies

Past calculations

- Formalisms: Frank & Rees 1976, Lightman & Shapiro 1977

  Hils & Bender 1995 - (LS formalism)

  Sigurdsson & Rees 1997, Sigurdsson 1998, 1999 (FR formalism + "kicks")

  Miralda-Escude & Gould 2000 (FR/BW), Ivanov 2002 (LS),

  Freitag 2000-2002 (time explicit Monte Carlo of FP, a la Hénon)

- Assume γ or p, scale to M_{BH} and one of σ, n_*, r_{BH}

  Evolved or unevolved stellar population (IMF, BH mass function).

  All assume isotropy and sphericity.

  Calculate relaxation.

- Benchmark rate to M32 rate.

  (M32 doesn't exist... cf Ziegler & Bender 1998)

  Find rates vary by $\sim 10^4$ depending on assumptions.

  Also rates are time variable - high rates deplete stellar population.

Comparisons and Uncertainties

- Hils & Bender find 1.8×10^{-8} per year for WD coalescence in M32

  cf Sigurdsson & Rees rate of 3×10^{-8} per year. Slightly lower diffusion rate, higher “kick” rate.

  Ivanov 3×10^{-9} per year - used high mass WDs, rate is consistent

  (SR comp. - integrated vs differential rate, structural parameters for M32).

  M-E & Gould - BH coalescence of $\sim 10^{-6}$ per year - (cf Sigurdsson & Rees estimate peak rate of $\sim 10^{-4}$ declining to 10^{-8}–10^{-6} per year.

  Sensitive to BH mass function and relaxation - M-E&G assume relaxed cusp, SR don’t.

  Freitag has dense isothermal cluster, with M-E & G IMF/BH-mf, finds somewhat higher but consistent rates (given assumptions).

- Unrelaxed, nucleated sub-L^* spirals dominate the rate

  very high coalescence rates are self-limiting - deplete population or grow BH to larger mass, coalescence rate declines with higher mass at fixed stellar density.

Aside - Check for runaway growth - find $\delta\theta_{lc}/\delta M \propto \theta_{lc}$, exponential growth but with very low growth constant - effectively linear growth.

Exception for high m_*/M_{BH} where $\delta\theta(\delta M)$ can be large and therefore approach runaway conditions.

Predictions

- All predictions consistent with LISA detections.

  Highest rate predictions imply possible source confusion.

  Uncertainties are primarily assumption driven, dominated by initial conditions assumed, not calculation method - in principle constrainable by observations.

  Some significant physics still not included.

- Direct capture vs spiral–in

  Gravitational radiation detection requires orbits that spiral in, captured into few r_S peribothrons ($e \sim 0.999 - 0.9999999$) but not directly into event horizon, or rapidly perturbed into event horizon.

  Estimates of direct loss fraction varies ($\sim 1/3$ or more???)

- Tidal disruption estimates:

  Similar processes apply to MS stars, should see tidal disruption events. Rate scales (but cf M-E & G), partially - r_{coll} for stars is much larger.

  Observationally, tidal disruption rate is consistent with maximum predicted rate Donley et al (2002) implies full loss-cone?

Future issues

- Brownian wandering

  Formalism assumes BH is fixed.

  In reality have $v_{BH}, \langle r_{BH} \rangle, t_{BH}$.

  If isothermal and relaxed, then trivial - $v_{iso-BH} = \sqrt{m_*/M_{BH}}\sigma$.

  - If $M_{BH} \approx m_*$ there is no loss cone.

  - If $M_{BH} \to \infty$ then loss cone is fixed.

  Black hole moves - what is mean free path, time scale and does it "carry the cusp with it" (adiabatic response of stars).

  Know that non-isothermal cusps in general have $v_{BH} = \eta v_{iso-BH}$, $\eta \gtrsim 1$ (cf Chatterjee et al 2002).

- Assume ballistic loss-cone exit.

$$t_{lc} \approx 10 r_S / \eta v_{iso-BH} \propto M_{BH}^{3/2}$$

For a $\sim 10^6 \mathrm{M}_\odot$ BH in a 100 km/sec cusp, we find $t_{lc} \sim 1/\eta 10^8$ seconds, or about 1 year. The time scale is longer if mean-free path is shorter (have to random walk out of loss-cone then).

...future wanderings...

Motion should be ballistic on time scales comparable to some fraction of cusp dynamical time, very roughly, $\sim 10^{2-3}$ years.

So, orbits with time scale less than 1 year are carried with the BH as it wanders. Which are orbits inside $\sim 100 AU$ for our canonical $10^6 \mathrm{M}_\odot$SMBH.

This is $\sim 10^4 r_S$, which is precisely where the dominant influx of compact objects is coming from.

Therefore BH wandering is significant, but possibly not dominant, with a critical mass close to $10^6 \mathrm{M}_\odot$ - more massive SMBH wander slowly and carry cusp with them, less massive SMBH leave loss-cone ballistically.

Critical to calculate η for non-isothermal cusps.

Also want the RMS displacement $\langle r_{BH} \rangle$,

as well as the mean free path r_{BH}

Black hole effectively fixed for when $r_{BH} \ll r_S$.

...future broken symmetries...

- Triaxiality

  If we can't move the SMBH, can we move the stars?

  If nucleus is triaxial, J is not conserved for individual stars on orbits - box orbits and boxlets walk through centre.

  close to BH, orbits are perturbed Keplerian orbits, precessing ellipses - transition inside r_h, how far?

- Scattering by SMBH leads to chaos

  ergodic orbits → spherical potential (cf Binney & Gerhard 1985)

  Not necessarily - Holley-Bockelmann et al (2002) - see figures

  Regular – regular scattering, allows continued centrophilic orbits.

  Hard problem, don't know what real galaxies do yet.

  Need to determine flux to low J from high E (large r). Time scales for evolution of potential shape (cf Zhao et al 2002).

- Mergers - refill loss cone on dynamical time scale for many cusp dynamical times. Most of the integrated rate may be post-merger.

  Coalescence during AGN phase. Disk variability signature?

...future formation

- Star Formation

  Largest outstanding uncertainty are the IMF, the BH mass function and whether star formation is ongoing...

- If Pop III IMF is peaked strongly to high mass, then high z, IMBH are strong contributors

- Need the zero mass cut-offs, frequency and distribution of masses for low mass black holes as function of IMF, Z

  binarity and initial spin would be nice too...

- If there is ongoing “normal” nuclear star formation in inner 1-2 pc of normal spirals, with as little as 1% duty cycle, then compact objects are replenished and high coalescence rate sustained (caveat BH mass function dependence on Z, see Fryer this meeting).

  Observational evidence (cf Eckart 1999, Ghez 2002)

  In situ formation, or off centre clusters? Need to understand formation channel and IMF

Conclusions

- Coalescence of low mass compact objects with SMBH is certain
- Rates are uncertain but consistent and optimistically high
- Outstanding uncertainties are resolvable in principle,

  good astrophysical motivation

  LISA will test models,

  some parameters free until LISA constrains them.