The detection and measurement of gravitational waves from periodic sources

Statistics for Gravitational Wave Data Analysis
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Talk overview

• What’s special about gravitational continuous wave signals?

• The relevance of coherence
• The good and the bad of narrow bandwidths

• Examples: the evolution of search methods and statistical approaches in the LSC CW Group
  – Targeted searches
  – Wide parameter searches

• Multi-detector methods
• Multi-source methods (LISA)

• Statistical conundrums, outstanding issues and personal opinions
The gravitational wave signal

- Take the signal as quasi-sinusoidal GWs from a triaxial, non-precessing neutron star, modulated by doppler motions and the antenna pattern of the GW detector:

\[ h(t) = \frac{1}{2} F_+ (t; \psi) h_0 (1 + \cos^2 \iota) \cos 2\Psi(t) + F_\times (t; \psi) h_0 \cos \iota \sin 2\Psi(t) \]

Signal model

\[ \Psi(T) = \phi_0 + 2\pi \left[ f_0 (T - T_0) + \frac{1}{2} \ddot{f}_0 (T - T_0)^2 + \frac{1}{6} \dddot{f}_0 (T - T_0)^3 + O(T^4) \right] \]

Signal phase

\[ T = t + \delta t = t + \Delta_{\text{Roemer}} + \Delta_{\text{Shapiro}} + \Delta_{\text{Einstein}} + \Delta_{\text{Binary}} \]

Barycentric corrections
The gravitational wave signal

• Model parameters are
  – 2 axial orientation angles \((\psi, \iota)\)
  – 1 signal amplitude \((h_0)\)
  – 2+ spin parameters \((\phi_0, f_0, df_0/dt, \ldots)\)
  – 2 sky location (RA, dec)
  – 4+ further parameters for binary sources

• Many parameters to seach/marginalise/maximise over, and some large dimensions (e.g., \(f\))

• However the signal is believed to be coherent on timescales of months to years, and \(~\)phase-locked to radio pulses (should they be available).
Coherence

- Continuous wave searches can go deeper (in strain) than any other GW search because the signal is deterministic and coherent
  - Coincident observations are not required or, for signal measurement, particularly helpful
  - Data from multiple detectors can be combined relatively simply
  - Follow-up observations are always possible

- With coherence comes a relatively narrow bandwidth...

Posterior pdf for strain amplitude from pulsar J1920-5959D. Quoted UL is $1.7 \times 10^{-24}$ (S2 data)
Bandwidth issues

• If we know the frequency and phase evolution of the signal (from radio observations) then robust, well-calibrated results can be obtained even in a generally poor noise and interference environment:

Signal frequency is clean

• Question: how do you know when it is clean enough?
Bandwidth issues

- But is the frequency is not known, there are many instrumental lines that can excite matched filters, making robust searches very difficult:

- Is sensitivity now defined by the strongest unidentified line, rather than the noise floor?

- Note, it’s unlikely that instrumental lines will exhibit the correct doppler/antenna pattern modulations so, in bayesian terms, evidence is more important than likelihood.
Search and measurement methods

- The LSC CW experience is that several, ideologically disparate, search methods can coexist very happily, and spark off each other:

Coherent searches:
- Time-domain:
  - Targeted
  - Markov Chain Monte Carlo

Incoherent searches:
- Hough transform
- Stack-Slide
- Powerflux

Searches over narrow parameter space (Bayesian)

Searches over wide parameter space

Excess power searches

Ultimately, would like to combine these two in a hierarchical scheme (frequentist, with some Bayesian leanings)
Statistical approaches

- **Targeted searches (t-domain Bayesian):**
  - Heterodyne at the expected signal frequency, accounting for spindown and doppler variations, then determine a marginal pdf for the strain amplitude (marginalise over all other model parameters, and the noise floor, every 30 min). Use Markov Chain Monte Carlo when numerically marginalising over more than 4 parameters.

- **Coherent wide area searches (f-domain, frequentist)**
  - Use a detection statistic ('F-statistic') which is the log likelihood, pre-maximised over $\iota$, $\phi$ and $\psi$. Incoherent combination of data from different detectors giving a frequentist UL based on the loudest coincident event.

- **Incoherent wide area searches (f-domain, frequentist)**
  - Stack and slide short power spectra then combine
    - after thresholding (Hough method)
    - after normalising (stack-slide method)
    - After weighting for the antenna pattern and noise floor (Powerflux method)
Coherent vs incoherent

• Clearly coherent searches always win out if computing time is not a constraint.

• But the trade-off is less obvious for a fixed computing time:
  – The size of the parameter space searched means that only big signals are statistically significant.
  – Incoherent methods are nearly as good as coherent methods once the signal is big!

• The trick is to integrate coherently for an optimal length of time, then combine these results incoherently, leading to hierarchical schemes:
  – Coherent sub-steps
  – Incoherent combination of coherent sub-steps
  – Fully coherent follow-up of candidates

A similar scheme is used in some radio pulsar searches.
Multi-detector analysis

- Within a Bayesian framework, multidetector (‘network’) analysis is particularly straightforward. The posterior for the model \( m \) is

\[
p(m \mid d_1d_2d_3) \propto p(m)p(d_1 \mid m)p(d_2 \mid m)p(d_3 \mid m)
\]

- But the results can be initially surprising - the joint upper limit \( \textit{can} \) be worse than some of the contributing individual upper limits (though this is rare).
Confusion - a future challenge for Bayes

- LISA data is expected to contain many (of order 50,000) signals from white dwarf binaries. The data will contain resolvable binaries and binaries that just contribute to the overall noise (either because they are faint or because their frequencies are too close together). How do we proceed?

- Bayes can sort these out without having to introduce ad-hoc acceptance and rejection criteria, and without needing to know the “true noise level” (whatever that means):
Confusion - a simple test passed

Test: The blind detection and parameterisation of 100 unknown sinusoids in noise of unknown variance (Umstatter et al., 2005)
Outstanding questions

• Bayesian:

  – It is common to use a uniform prior for the signal amplitude, $h_0$. This has some attractions for upper limit work, but gives misleading intervals. Can we construct a non-informative proper prior for $h_0$, or perhaps for $(h_0 \cos \phi, h_0 \sin \phi$)? I think not - we should never be able to construct a posterior interval (based on an non-informative prior) that excludes $h_0=0$ at high probability when the data are consistent with $h_0=0$.

  – Should we use the parameter estimation posterior $p(h_0 | \text{data})$ to express our sensitivity to a signal, or should we use a measure based on Bayesian evidence (in the sense of model likelihood)?

  – How do we use MCMC methods effectively when there is no signal?
Outstanding questions

- **Frequentist**
  - How do we tackle instrumental lines in a wide-area search? Is an upper-limit result based on the loudest coincident event between detectors the best we can do?

  - Statistics (like the $F$ statistic) allow for a very efficient parameter space search, but a very clumsy final statement on sensitivity. Can/should we recast the statistic in Bayesian terms (i.e., use the statistic as the data in an inference statement)?

  - Is the ‘worst case scenario’ parameter combination a sensible way to set upper limits? [For me, the worst case scenario is that the noise has conspired to exactly cancel out the signal!]