Characterising confusion: Inference from stochastic GW searches

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Outline

• Stochastic GWs
• Search method
• Statistical analyses
• Other searches / Future plans

NOTE:
  - All-sky IFO-IFO search using LHO-LLO
  - Methods talk / Signal identification
Stochastic GWs

- Random GW signal produced by a large number of weak, independent, unresolved GW sources
- Detect by cross-correlating two GW detectors
- Strength specified by the ratio of energy in GWs to total energy density needed to close universe
Cross-correlation strategy

- Spectrum:
  \[ \frac{\rho_{GW}}{\rho_{critical}} = \int_0^\infty d(\ln f) \Omega_{GW}(f) \]

- Cross-correlation statistic:
  \[ Y \sim \int df \ s_1(f) Q(f) s_2(f) \]

- Optimal filter:
  \[ Q(f) \propto \frac{\gamma(f) \Omega_{GW}(f)}{f^3 P_1(f) P_2(f)} \quad \text{where} \quad \Omega_{GW}(f) = \Omega_0 (f/100 \text{Hz})^\alpha \]

- Mean (Y) = \[ \Omega_0 \]
  \[ \text{Var} (Y) \equiv \sigma_Y^2 \propto \frac{1}{T} \left[ \int df \ \frac{\gamma^2(f)}{f^6 P_1(f) P_2(f)} \right]^{-1} \]
Example: Overlap Reduction Function (LLO and other detectors)

\[ \gamma(f) \]

- Red: LHO
- Brown: GEO-500
- Blue: Virgo
- Green: ALLEGRO

Frequency (Hz)

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Analysis details

- Split data into short segments (e.g., 60 sec):
  \[ \{Y_I, \sigma_{Y_I}\}, I = 1, 2, \ldots \]
- Down-sample (16384Hz to 1024Hz)
- HP filter & window to reduce spectral leakage
- Calibrate data in frequency domain
- Sliding PSD estimation to eliminate bias
- Remove coherent lines (n*16Hz, n*60Hz)
- Optimal filter for different spectral indices \( \alpha \)
- Frequency band with 99% sensitivity
Long-term coherence (H1-L1)
Long-term coherence (H1-H2)

Coherence level for uncorrelated time-series

Broad band coherence (acoustic coupling)
Bayesian analysis

- Likelihood:
  \[ p(\{Y_I\}, \Omega_0, \{\sigma_Y\}) \propto \exp\left[-\frac{1}{2} \frac{(Y_{opt} - \Omega_0)^2}{\sigma_{opt}^2}\right] \]

- Prior:
  \[ p(\Omega_0) = \frac{1}{\Omega_{max}} \text{ for } 0 < \Omega_0 < \Omega_{max} \]

- Posterior:
  \[ p(\Omega_0: \{Y_I\}, \{\sigma_Y\}) \propto \exp\left[-\frac{1}{2} \frac{(Y_{opt} - \Omega_0)^2}{\sigma_{opt}^2}\right] \text{ for } 0 < \Omega_0 < \Omega_{max} \]
90% probability ULs

\[ 0.90 = \int_{0}^{\Omega_{0}^{UL}} d\Omega_{0} p(\Omega_{0} : \{Y_{I}\}, \{\sigma_{Y_{I}}\}) \]

\[ \Omega_{0}^{UL} = 3.29 \sigma_{opt} \]

\[ \Omega_{max} = 5 \sigma_{opt} \]

\[ Y_{opt} = 2 \sigma_{opt} \]
90% probability ULs

\[ Y_{\text{opt}} = -\sigma_{\text{opt}} \]
\[ \Omega_{\text{max}} = 5 \sigma_{\text{opt}} \]

\[ \Omega_0^{\text{UL}} = 1.15 \sigma_{\text{opt}} \]
Optimal combination of $Y_I, \sigma_{Y_I}$

- Non-overlapping windows:
  \[
  \frac{Y_{opt}}{\sigma_{opt}^2} = \sum_I Y_I / \sigma_{Y_I}^2 \\
  \frac{1}{\sigma_{opt}^2} = \sum_I 1 / \sigma_{Y_I}^2
  \]

- 50% overlapping windows:
  - Correlations between adjacent segments $Y_I, Y_{I+1}$
  - Odd (even) segments are not correlated with another
  - Combine $Y_{odd}$ and $Y_{even}$ using covariance matrix
  - LIGO- T040089-00-Z
50% overlapping windows

A.  
\[ \text{I=1} \quad \text{I=3} \quad \ldots \ldots \ldots \quad \text{I=M-2} \quad \text{I=M} \]

\[ \{x_i\}, i=1 \ldots R \]

B.  
\[ \text{x}_1 \quad \text{w}_i, \quad \text{y}_i, \quad \sigma_{x_1}^2 \quad \text{x}_N \]

\[ \{x_i\}, i=1 \ldots 3N/2 \]

\[ \text{N/2} \]

\[ \text{w}_{i+1}, \quad \text{y}_{i+1}, \quad \sigma_{x(i+1)}^2 \]
Optimal combination

\[ Y_{opt} / \sigma_{opt}^2 = \sum_{a,b} ||C^{-1}||_{ab} Y_b \]

\[ 1 / \sigma_{opt}^2 = \sum_{a,b} ||C^{-1}||_{ab} \]

where

\[ ||C|| = \begin{pmatrix} \sigma_{odd}^2 & \sigma_{odd, even}^2 \\ \sigma_{odd, even}^2 & \sigma_{even}^2 \end{pmatrix} \]

\[ Y_{odd} / \sigma_{odd}^2 = \sum_{I_{odd}} Y_I / \sigma_{Y,I}^2 \]

\[ 1 / \sigma_{odd}^2 = \sum_{I_{odd}} 1 / \sigma_{Y,I}^2 \]

\[ Y_{even} / \sigma_{even}^2 = \sum_{I_{even}} Y_I / \sigma_{Y,I}^2 \]

\[ 1 / \sigma_{even}^2 = \sum_{I_{even}} 1 / \sigma_{Y,I}^2 \]

\[ \sigma_{odd, even}^2 \approx \frac{1}{2} (\sigma_{odd}^2 + \sigma_{even}^2) \frac{3}{35} \left(1 - 1/M\right) \]

Hann windows

End effect (M~50)
Marginalise over CC instr noise

- Measure 'effective' correlation: $\Omega_0 + \Omega_{\text{inst}}$

- Likelihood:

$$p(\{Y_I\}, \Omega_0, \Omega_{\text{inst}}, \{\sigma_{Y_I}\}) \propto \exp[-\frac{1}{2} \frac{(Y_{opt} - \Omega_0 - \Omega_{\text{inst}})^2}{\sigma^2_{\text{opt}}} ]$$

- Priors:

$$p(\Omega_0) = \frac{1}{\Omega_{\text{max}}} \text{ for } 0 < \Omega_0 < \Omega_{\text{max}}$$

$$p(\Omega_{\text{inst}}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{inst}}} \exp[-\frac{1}{2} \frac{(\Omega_{\text{inst}} - \Omega_{\text{inst}, 0})^2}{\sigma^2_{\text{inst}}} ]$$

- Posterior: (for $0 < \Omega_0 < \Omega_{\text{max}}$)

$$p(\Omega_0; \{Y_I\}, \{\sigma_{Y_I}\}) \propto \exp[-\frac{1}{2} \frac{(Y_{opt} - \Omega_0 - \Omega_{\text{inst}, 0})^2}{\sigma^2_{\text{opt}} + \sigma^2_{\text{inst}}} ]$$
Marginalise over theor variance

- $\{Y_I\}_\alpha \sim N_\alpha$ values such that $\sigma_I \approx const = \sigma_\alpha$

- Likelihood:

$$p(\{Y_I\}_\alpha : \Omega_0, \sigma_\alpha) \propto \frac{1}{\sigma^{N_\alpha}_\sigma} \exp\left[-\frac{1}{2} \frac{(Y_{opt,\alpha} - \Omega_0)^2 + s_{\sigma}^2}{\sigma^{2}/N_\alpha}\right]$$

where

$$Y_{opt,\alpha} = \frac{1}{N_\alpha} \sum_{I \in \alpha} Y_I$$

$$s_{\sigma}^2 = \frac{1}{N_\alpha} \sum_{I \in \alpha} (Y_I - Y_{opt,\alpha})^2$$

- Priors:

$$p(\Omega_0) = 1/\Omega_{\max} \quad \text{for} \quad 0 < \Omega_0 < \Omega_{\max}$$

$$p(\sigma_\alpha) = 1/\sigma_\alpha \quad \text{Jeffrey's prior}$$

- Posterior: (for $0 < \Omega_0 < \Omega_{\max}$)

$$p(\Omega_0 : \{Y_I\}_\alpha) \propto \left[(Y_{opt,\alpha} - \Omega_0)^2 + s_{\sigma}^2\right]^{-N_\alpha/2}$$
Normalised residuals

Cumulative PDF for normalised residuals (S3 H1–L1)

\[ x_I = \frac{(Y_I - Y_{opt})}{\sigma_{Y_I}} \]
Other searches / Future plans

• On-going:
  - S2 ALLEGRO-LLO nearly complete
  - S3 H1-H2 analysis??
  - S4 targeted search for anisotropic stochastic GWs

• Near future:
  - S4 analysis for arbitrary $\Omega_{GW}(f)$ (U B'ham)
  - High freq IFO-IFO search (H1-H2 at 37.52 kHz)