



Resolving continuous sources with ground-based broad-band detectors

Ian Jones

`jones@gravity.psu.edu`

Penn State Center for Gravitational Wave Physics



Outline

- Types of source



Outline

- Types of source
- Resolution in frequency



Outline

- Types of source
- Resolution in frequency
- Resolution in angular position



Outline

- Types of source
- Resolution in frequency
- Resolution in angular position
- Resolution in distance



Outline

- Types of source
- Resolution in frequency
- Resolution in angular position
- Resolution in distance
- Gravitational wave amplitudes



Outline

- Types of source
- Resolution in frequency
- Resolution in angular position
- Resolution in distance
- Gravitational wave amplitudes
- What can we learn about the clusters?



Outline

- Types of source
- Resolution in frequency
- Resolution in angular position
- Resolution in distance
- Gravitational wave amplitudes
- What can we learn about the clusters?
- What can we learn about the neutron stars?



Outline

- Types of source
- Resolution in frequency
- Resolution in angular position
- Resolution in distance
- Gravitational wave amplitudes
- What can we learn about the clusters?
- What can we learn about the neutron stars?
- Open questions



Types of source

The only types of continuous source which lie in the LIGO frequency band are *spinning neutron stars*. Two sorts:

- Millisecond pulsars
 - Gravitational wave emission caused by triaxiality
 $\Rightarrow f_{\text{GW}} = 2f_{\text{spin}}$.
 - ≈ 56 known in GCs, but actual number much greater.
- LMXBs
 - Gravitational wave emission caused by triaxiality
 $\Rightarrow f_{\text{GW}} = 2f_{\text{spin}}$ or by an r-mode $\Rightarrow f_{\text{GW}} = 4/3f_{\text{spin}}$.
 - 13 known in GCs. Probably complete.



Frequency resolution

The frequency resolution of a coherent analysis is at least as good as $\delta f \sim 1/T_{\text{obs}}$:

$$\delta f \sim 3.2 \times 10^{-8} \text{ Hz} \left(\frac{1 \text{ year}}{T_{\text{obs}}} \right)$$

$$\delta f \sim 1.2 \times 10^{-5} \text{ Hz} \left(\frac{1 \text{ day}}{T_{\text{obs}}} \right)$$

Number of MSP in Galaxy of order 10^5
 \Rightarrow no danger of confusion in frequency.



Angular resolution

Doppler shift induced by orbital motion $\sim f\Omega R/c$. For two points on sky separated by angle $\delta\theta$, *difference* in Doppler shifts is

$$\delta f_{\text{Doppler}} \sim f \frac{\Omega R}{c} \delta\theta$$

Uncertainty in extracting frequency from noisy data stream is

$$\delta f_{\text{data}} \sim \frac{A}{T_{\text{obs}}\rho}$$

Combine to give angular resolution:

$$\delta\theta \sim \frac{Ac}{f\Omega RT_{\text{obs}}\rho}$$



Angular resolution cont...

$$\Rightarrow \delta\theta \sim 1.3 \times 10^{-7} \text{ radians} \left(\frac{\text{kHz}}{f} \right) \left(\frac{1 \text{ year}}{T_{\text{obs}}} \right) \left(\frac{10}{\rho} \right) \left(\frac{A}{4} \right)$$

Compare with size of a globular cluster:

$$\delta\theta_{\text{cluster}} \sim 2 \times 10^{-4} \text{ radians} \left(\frac{r_{\text{core}}}{1 \text{ pc}} \right) \left(\frac{5 \text{ kpc}}{d} \right)$$

\Rightarrow typical globular cluster well resolved.

Need $\sim 10^6$ patches for optimal data analysis.



Distance resolution

GW amplitude scales as $h \sim \epsilon f^2/d$, where $\epsilon \sim \delta I/I$.

Can measure h and f . Setting $d = d_{GC} \Rightarrow \epsilon$.

If $\epsilon > \epsilon_{\max}$ can reject association, but ϵ_{\max} highly uncertain.

Use a statistical argument instead:

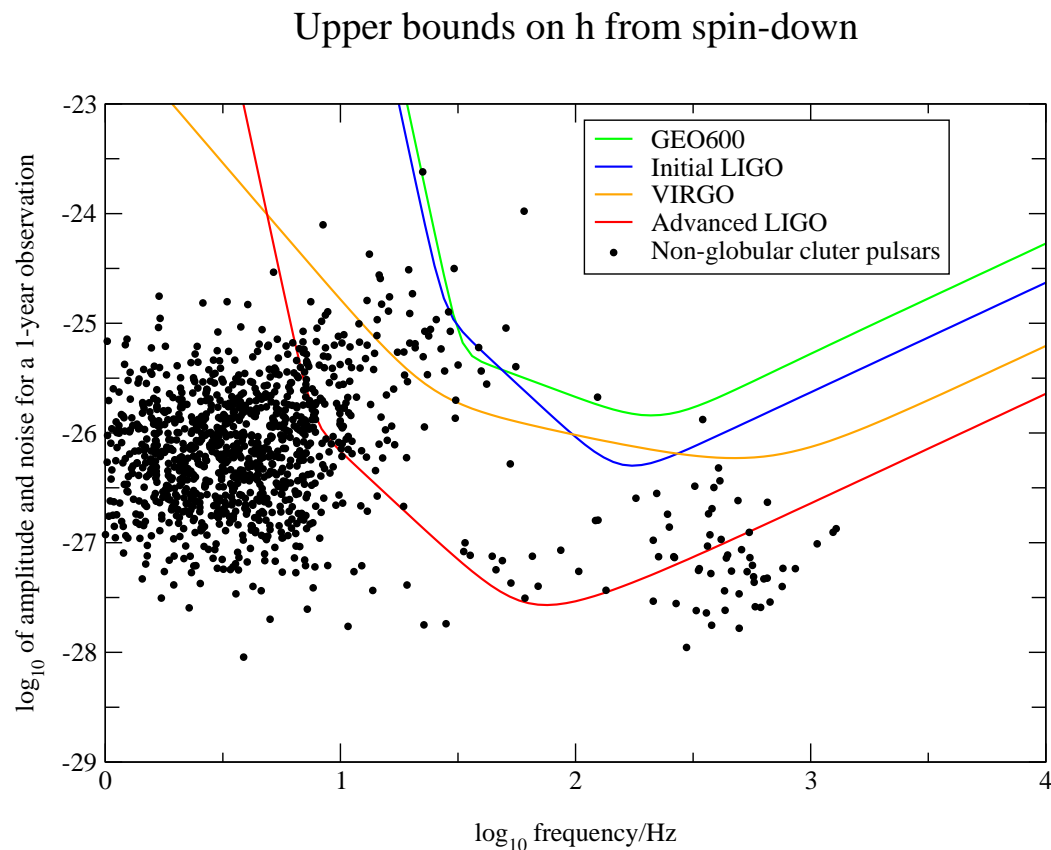
$$\delta N \sim R^3 \delta\theta^2 \sim 10^{-2} \times \left(\frac{n}{10^3 / \text{kpc}^3} \right) \left(\frac{R}{10 \text{ kpc}} \right)^3 \left(\frac{\delta\theta}{10^{-4} \text{ radians}} \right)^2$$

\Rightarrow chance of spurious alignment small.



Gravitational wave amplitudes

Can make a naive guess by assuming all spin-down energy goes into gravitational wave energy.

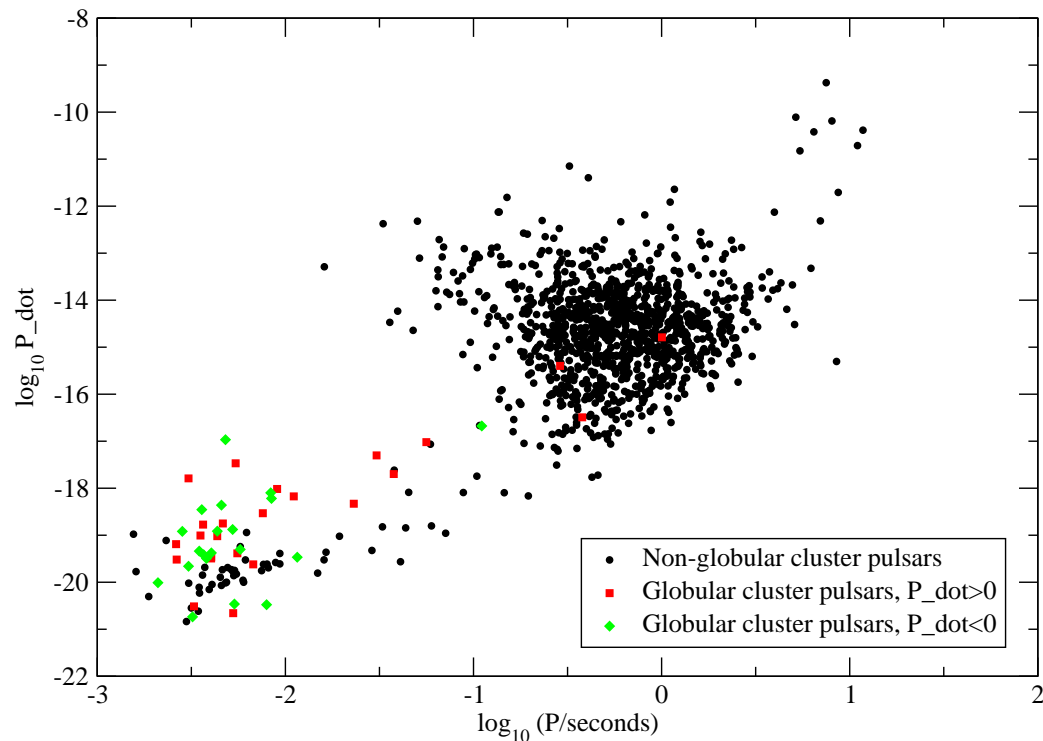




Gravitational wave amplitudes cont...

The $P - \dot{P}$ diagram shows that such a procedure is clearly unsafe for GC pulsars:

P-P_dot diagram for all known pulsars





Gravitational wave amplitudes cont...

This is easily understood:

$$\dot{f}_{\text{spin-down}} \sim \frac{f}{\tau} \sim 3 \times 10^{-14} \text{ Hz/s} \left(\frac{f}{\text{kHz}} \right) \left(\frac{10^9 \text{ years}}{\tau} \right)$$

$$\dot{f}_{\text{Doppler}} \sim f \frac{a}{c} \sim 5 \times 10^{-13} \text{ Hz/s} \left(\frac{M}{10^6 M_{\odot}} \right) \left(\frac{1 \text{ pc}}{x} \right)^2 \left(\frac{f}{\text{kHz}} \right)$$

⇒ globular cluster potential can dominate intrinsic spin-down.



Gravitational wave amplitudes cont...

Complications

- **Selecting known pulsars for targeting:** Knowledge of $d, f, \dot{f}, h_{\text{detector}}$ allows estimate of whether source is detectable or not.
- **Searching for unknown pulsars:** Need to search over large number of \dot{f} values to ensure coherence maintained over observation:

$$N \sim 3 \times 10^2 \left(\frac{\dot{f}_{\text{max}}}{10^{-13} \text{ Hz}} \right) \left(\frac{T_{\text{obs}}}{1 \text{ year}} \right)^2 \left(\frac{1 \text{ radian}}{\Delta\Phi} \right)$$

- **Interpretation:** Measurement of $h, f, d \Rightarrow \epsilon \Rightarrow \dot{f}_{\text{GW}}$.
Then

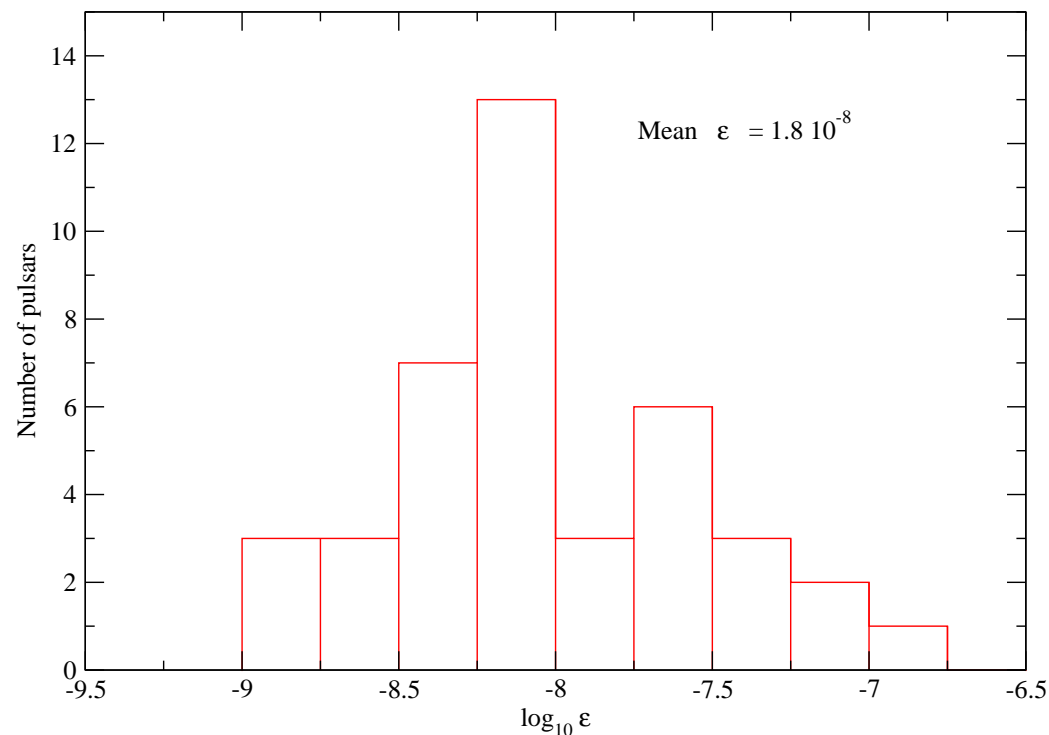
$$\dot{f}_{\text{observed}} = \dot{f}_{\text{GW}} + \dot{f}_{\text{EW}} + \dot{f}_{\text{acceleration}}$$



Gravitational wave amplitudes cont...

In absence of trustworthy spin-down rates we can use the distribution of ϵ values for non-GC pulsars as a guide:

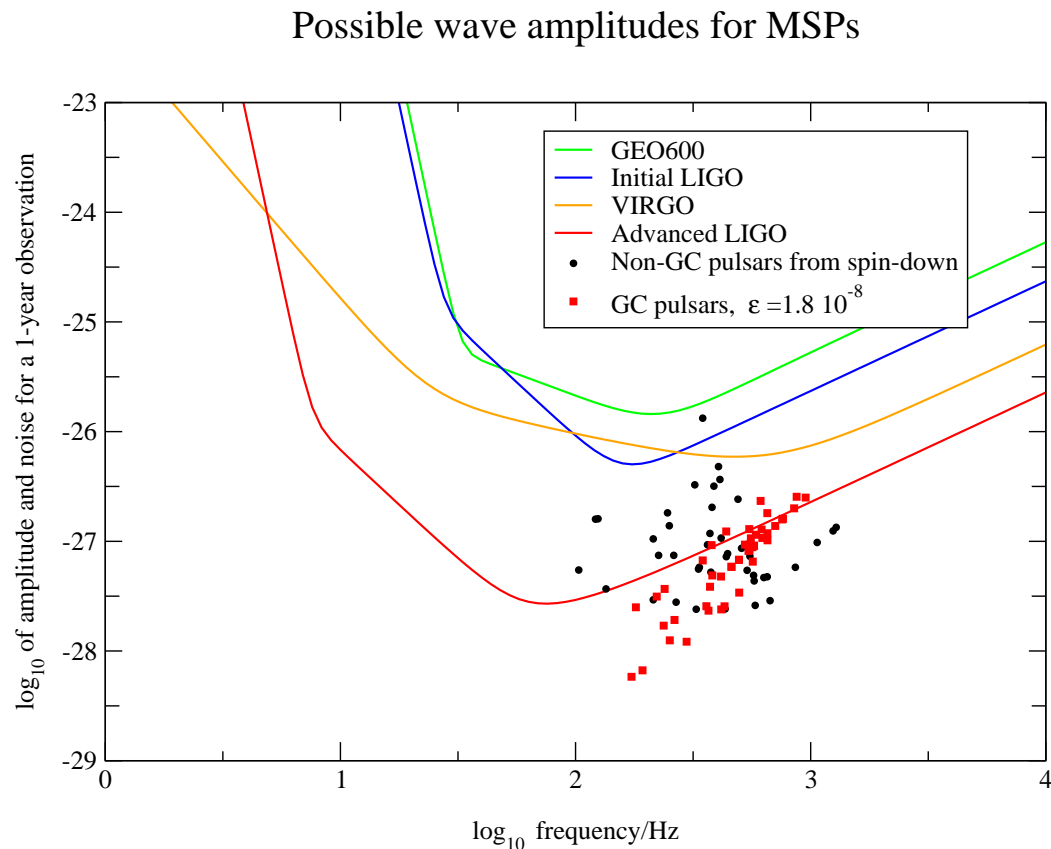
Distribution of ϵ values for non-GC pulsars





Gravitational wave amplitudes cont...

Taking the mean ϵ value computed for non-GC pulsars to apply to the GC ones:

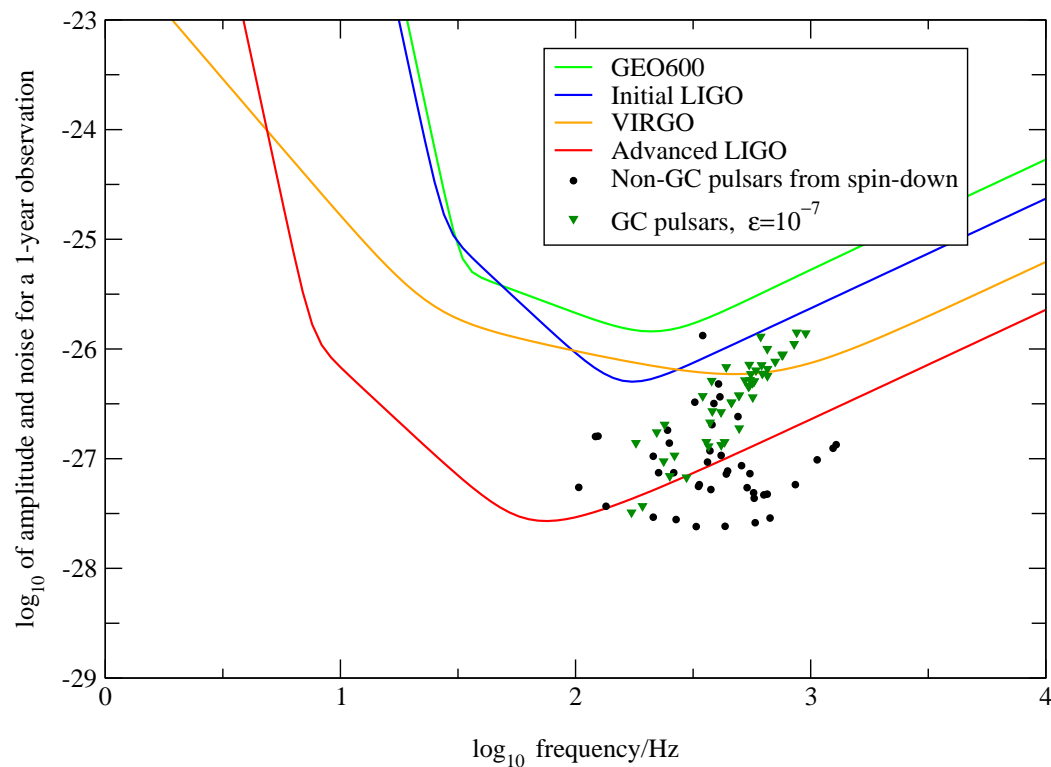




GW amplitudes for MSPs

Taking a larger value from the edge of the non-GC distribution:

Possible wave amplitudes for MSPs





Recap...



What can we learn about the cluster?

- Improved estimate of number of neutron stars retained by cluster.
- High resolution map of location of neutron stars.
- For some (most?) we can read off their accelerations.
- Combination of above with dynamical models \Rightarrow density profile of cluster.



What can we learn about the neutron stars?

- Magnitude of triaxiality of MSP \Rightarrow breaking strain of crust.
- Magnitude of triaxiality of LMXB \Rightarrow breaking strain of crust plus indication Bildsten mechanism may be operative.
- Magnitude of r-mode \Rightarrow indication that instability isn't killed by viscosity and probe of non-linear dynamics.

One further point **unique to GCs**:

- Collection of stars all at same distance \Rightarrow precision test of uniformity of triaxiality/r-mode saturation amplitude.



Open questions:

1. How long will it actually take to search 1 year's worth of data for a cluster?
2. How useful will accurate neutron star positions/acceleration be for GC modelers? Are the simulations in place to interpret the new data?
3. Do we understand cluster dynamics well enough to be able to say that accelerations unimportant for some neutron stars?
4. Could GW observations reveal new clusters?